Chapter 1 Review Exercises

In Exercises 1-4, match the data with the real-life situation that it represents. (Graphs are labeled (a)-(d).)

(a)  
(b)  
(c)  
(d)  

1. Population of Texas  
2. Population of California  
3. Number of U.S. business failures  
4. IBM revenues

In Exercises 5-8, find the distance between the two points.
5. (0, 0), (5, 2)  
6. (1, 2), (4, 3)  
7. (−1, 3), (−4, 6)  
8. (6, 8), (−3, 7)

In Exercises 9-12, find the midpoint of the segment connecting the two points.
9. (5, 6), (9, 2)  
10. (0, 0), (−4, 8)  
11. (−10, 4), (−6, 8)  
12. (7, −9), (−3, 5)

In Exercises 13 and 14, use the graph below, which gives the revenues, costs, and profits for Kimberly-Clark Corporation from 1991 through 1996. (Source: Kimberly-Clark Corporation)

Kimberly-Clark Corporation

13. Which bars on the graph represent the revenues? Which represent the costs? Which represent the profits? Explain your reasoning. Write an equation that relates the revenue $R$, cost $C$, and profit $P$.

14. Estimate the revenue, cost, and profit for Kimberly-Clark Corporation in 1996. Do you think these numbers are in billions of dollars, millions of dollars, or thousands of dollars? Explain your reasoning. (Kimberly-Clark is the maker of Kleenex, Huggies Disposable Diapers, and other paper products.)

15. Translate the triangle whose vertices are (1, 3), (2, 4), and (5, 6) three units to the right and four units up. Find the coordinates of the translated vertices.

16. Biology: Species Collection The following data represent six intertidal invertebrate species collected from four stations along the Maine coast.

\[
\begin{array}{ccc}
\text{Mytilus} & 107 & \text{Gammara} & 78 \\
\text{Littorina} & 65 & \text{Arbacia} & 6 \\
\text{Nassarius} & 112 & \text{Mya} & 18 \\
\end{array}
\]

Use a graphing utility to construct a bar graph that represents the data. (Source: Adapted from Haefner, Exploring Marine Biology: Laboratory and Field Exercises)
In Exercises 17–24, sketch the graph of the equation.
17. \( y = 4x - 12 \)
18. \( y = 4 - 3x \)
19. \( y = x^2 + 5 \)
20. \( y = 1 - x^2 \)
21. \( y = x^2 + 5x + 6 \)
22. \( y = |2x - 3| \)
23. \( y = x^3 + 4 \)
24. \( y = \sqrt{x} \)

In Exercises 25 and 26, find the intercepts of the graph of the equation algebraically. Check your results with a graphing utility.
25. \( 4x + y + 3 = 0 \)
26. \( y = (x - 1)^3 + 2(x - 1)^2 \)

In Exercises 27 and 28, write the standard form of the circle.
27. Center: (0, 0); Solution point: \((2, \sqrt{5})\)
28. Center: \((2, -1)\); Solution point: \((-1, 7)\)

In Exercises 29 and 30, complete the square to write the equation of the circle in standard form. Determine the radius and center of the circle. Then sketch the circle.
29. \( x^2 + y^2 - 6x + 8y = 0 \)
30. \( x^2 + y^2 + 10x + 4y - 7 = 0 \)

In Exercises 31–34, find the points of intersection of the graphs algebraically. Then use a graphing utility to check your results.
31. \( x + y = 2, \ 2x - y = 1 \)
32. \( x^2 + y^2 = 5, \ x - y = 1 \)
33. \( y = x^3, \ y = x \)
34. \( y = \sqrt{x}, \ y = x \)

35. Break-Even Analysis: The student government association wants to raise money by having a T-shirt sale. Each shirt costs $8. The silk screening costs $200 for the design, plus $2 per shirt. Each shirt will sell for $14.
(a) Find equations for the total cost \( C \) and the total revenue \( R \) for selling \( x \) shirts.
(b) Find the break-even point.

36. Break-Even Analysis: You are starting a part-time business. You make an initial investment of $6000. The unit cost of the product is $6.50, and the selling price is $13.90.
(a) Find equations for the total cost \( C \) and the total revenue \( R \) for selling \( x \) units of the product.
(b) Find the break-even point.

In Exercises 37–42, find the slope and \( y \)-intercept (if possible) of the linear equation. Then sketch the graph of the equation.
37. \( 3x + y = -2 \)
38. \(-\frac{1}{4}x + \frac{3}{4}y = 1\)
39. \( y = \frac{5}{3} \)
40. \( x = -3 \)
41. \( -2x - 5y = 5 \)
42. \( 3.2x - 0.8y + 5.6 = 0 \)

In Exercises 43–46, find the slope of the line passing through the two points.
43. \((0, 0), (7, 6)\)
44. \((-1, 5), (-5, 7)\)
45. \((10, 17), (-11, -3)\)
46. \((-11, -3), (-1, -3)\)

In Exercises 47 and 48, find an equation of the line that passes through the point and has the indicated slope. Then use a graphing utility to graph the line.
47. Point: \((3, -1)\); Slope: \(m = -2\)
48. Point: \((-3, -3)\); Slope: \(m = \frac{1}{3}\)

In Exercises 49 and 50, find the general form of the equation of the line passing through the point and satisfying the given condition.
49. Point: \((-3, 6)\)
(a) Slope is \(\frac{2}{3}\).
(b) Parallel to the line \(4x + 2y = 7\).
(c) Passes through the origin.
(d) Perpendicular to the line \(3x - 2y = 2\).
50. Point: \((1, -3)\)
(a) Parallel to the \(x\)-axis.
(b) Perpendicular to the \(x\)-axis.
(c) Parallel to the line \(-4x + 5y = -3\).
(d) Perpendicular to the line \(5x - 2y = 3\).

51. Demand: When a wholesaler sold a product at $32 per unit, sales were 750 units per week. After a price increase of $5 per unit, however, the sales dropped to 700 units per week.
(a) Write the quantity demanded \( x \) as a linear function of the price \( p \).
(b) Linear Interpolation: Predict the number of units sold at a price of $34.50 per unit.
(c) Linear Extrapolation: Predict the number of units sold at a price of $42.00 per unit.
52. **Linear Depreciation** A small business purchases a typesetting system for $117,000. After 9 years, the system will be obsolete and have no value.

(a) Write a linear equation giving the value $y$ of the system in terms of the time $t$.
(b) Use a graphing utility to graph the function.
(c) Use a graphing utility to estimate the value of the system after 4 years.
(d) Use a graphing utility to estimate the time when the system’s value will be $84,000.

In Exercises 53–56, use the vertical line test to determine whether $y$ is a function of $x$.

53. $y = -x^2 + 2$

54. $x^2 + y^2 = 4$

55. $y^2 - \frac{1}{4}x^2 = 4$

56. $y = |x + 4|$}

57. Given $f(x) = 3x + 4$, find the following.

(a) $f(1)$

(b) $f(x + 1)$

(c) $f(2 + \Delta x)$

58. Given $f(x) = x^2 + 4x + 3$, find the following.

(a) $f(0)$

(b) $f(x - 1)$

(c) $f(x + \Delta x) - f(x)$

In Exercises 59–64, use a graphing utility to graph the function. Then find the domain and range of the function.

59. $f(x) = x^2 + 3x + 2$

60. $f(x) = 2$

61. $f(x) = \sqrt{x + 1}$

62. $f(x) = \frac{x - 3}{x^2 + x - 12}$

63. $f(x) = -|x| + 3$

64. $f(x) = -\frac{12}{5}x - \frac{7}{8}$

In Exercises 65 and 66, use $f$ and $g$ to find the following.

(a) $f(x) + g(x)$

(b) $f(x) - g(x)$

(c) $f(x)g(x)$

(d) $\frac{f(x)}{g(x)}$

(e) $f(g(x))$

(f) $g(f(x))$

65. $f(x) = 1 + x^2, \quad g(x) = 2x - 1$

66. $f(x) = 2x - 3, \quad g(x) = \sqrt{x + 1}$

In Exercises 67–70, find the inverse of $f$ (if it exists).

67. $f(x) = \frac{3}{2}x$

68. $f(x) = |x + 1|$

69. $f(x) = -x^2 + \frac{1}{2}$

70. $f(x) = x^3 - 1$

In Exercises 71–86, find the limit (if it exists).

71. $\lim_{x \to 2} (5x - 3)$

72. $\lim_{x \to 2} (2x + 9)$

73. $\lim_{x \to 2} (5x - 3)(2x + 3)$

74. $\lim_{x \to 2} \frac{5x - 3}{2x + 9}$

75. $\lim_{x \to 2} \frac{x^2 + 1}{x}$

76. $\lim_{x \to 1} \frac{1}{x - 2}$

77. $\lim_{x \to 0} \frac{x^2 + 1}{x}$

78. $\lim_{x \to 2} \frac{1}{x - 2}$

79. $\lim_{x \to 3} \frac{x + 2}{x^2 - 4}$

80. $\lim_{x \to 2} \frac{x^2 - 9}{x - 3}$

81. $\lim_{x \to 0} \frac{1}{x - 1}$

82. $\lim_{x \to 2} \frac{2x - 1}{6x - 3}$

83. $\lim_{x \to 0} \left[ \frac{1}{x - 2} \right] - 1$

84. $\lim_{x \to 6} \frac{1}{x + 1} - 1$

85. $\lim_{x \to 0} \frac{(x + \Delta x)^2 - (x + \Delta x) - (x^2 - x)}{\Delta x}$

86. $\lim_{x \to 0} \frac{1 - (x + \Delta x)^2}{\Delta x} - (1 - x^2)$
In Exercises 87 and 88, use a table to estimate the limit.

87. \( \lim_{{x \to 0}} \frac{2x + 1 - \sqrt{3}}{x - 1} \)  
88. \( \lim_{{x \to 0}} \frac{1 - \sqrt{x}}{x - 1} \)

In Exercises 89–94, determine whether the statement is true or false.

89. \( \lim_{{x \to 0}} \frac{|x|}{x} = 1 \)  
90. \( \lim_{{x \to 0}} x^3 = 0 \)  
91. \( \lim_{{x \to 0}} \sqrt{x} = 0 \)  
92. \( \lim_{{x \to 0}} \sqrt{x} = 0 \)

93. \( \lim_{{x \to 2}} f(x) = 3, \quad f(x) = \begin{cases} 
3, & x \leq 2 \\
0, & x > 2
\end{cases} \)

94. \( \lim_{{x \to 3}} f(x) = 1, \quad f(x) = \begin{cases} 
-2, & x \leq 3 \\
-x^2 + 8x - 14, & x > 3
\end{cases} \)

In Exercises 95–102, find the intervals on which the function is continuous.

95. \( f(x) = \frac{1}{(x + 4)^2} \)  
96. \( f(x) = \frac{x + 2}{x} \)

97. \( f(x) = \frac{3}{x + 1} \)  
98. \( f(x) = \frac{x + 1}{2x + 2} \)

99. \( f(x) = \lceil x + 3 \rceil \)  
100. \( f(x) = \frac{3x^2 - x - 2}{x - 1} \)

101. \( f(x) = \begin{cases} 
x, & x \leq 0 \\
x + 1, & x > 0
\end{cases} \)

102. \( f(x) = \begin{cases} 
x, & x \leq 0 \\
x^2, & x > 0
\end{cases} \)

In Exercises 103 and 104, find \( a \) such that \( f \) is continuous on \((-\infty, \infty)\).

103. \( f(x) = \begin{cases} 
-x + 1, & x \leq 3 \\
ax^2 - 8, & x > 3
\end{cases} \)

104. \( f(x) = \begin{cases} 
x + 1, & x < 1 \\
2x + a, & x \geq 1
\end{cases} \)

105. **National Debt** The table lists the national debt \( D \) (in billions of dollars) for selected years. A mathematical model for the national debt is

\[ D = 0.2t^3 + 3.05t^2 + 5.88t + 387.94, \]

where \( t = 0 \) represents 1970.  (Source: U.S. Office of Management and Budget)

### Table for 105

<table>
<thead>
<tr>
<th>( t )</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D )</td>
<td>381</td>
<td>542</td>
<td>914</td>
<td>1817</td>
</tr>
<tr>
<td>( D )</td>
<td>3266</td>
<td>3599</td>
<td>4083</td>
<td>4436</td>
</tr>
</tbody>
</table>

(a) Use a graphing utility to graph the model.
(b) Create a table that compares the values given by the model with the actual data.
(c) Use the model to estimate the national debt in 1999.

106. **Recycling** A recycling center pays $0.25 for each pound of aluminum cans. Twenty-four aluminum cans weigh 1 pound. A mathematical model for the amount paid \( A \) by the recycling center is

\[ A = \frac{1}{4} \left\lfloor \frac{x}{24} \right\rfloor, \]

where \( x \) is the number of cans.

(a) Use a graphing utility to graph the function and then discuss its continuity.

(b) How much does a recycling center pay out for 1500 cans?

107. **Biology: Cross Fertilization** A researcher experimenting with strains of corn produced the results in the figure below. From the figure, visually estimate the \( x \)- and \( y \)-intercepts, and use these points to write an equation for each line.

(Source: Adapted from Levine/Miller, *Biology: Discovering Life, Second Edition*)
Chapter 2  Review Exercises

In Exercises 1–4, approximate the slope of the tangent line to the graph at \((x, y)\).

1. \[ \text{Graph 1} \]

2. \[ \text{Graph 2} \]

3. \[ \text{Graph 3} \]

4. \[ \text{Graph 4} \]

5. Revenue  The graph approximates the annual revenue (in millions of dollars per year) of Reebok, International for the years 1990–1996, with \(t = 0\) corresponding to 1990. Estimate the slope of the graph when \(t = 2\) and \(t = 4\). Interpret each slope in the context of the problem.  (Source: Reebok, International)

6. Cellular Telephones  The graph approximates the number of subscribers (in thousands per year) of cellular telephones for 1988–1995, with \(t = 0\) corresponding to 1988. Estimate the slope of the graph when \(t = 2\) and \(t = 6\). Interpret each slope in the context of the problem.  (Source: Cellular Telecommunications Industry Association)

In Exercises 7–10, use the limit definition to find the slope of the tangent line to the graph of \(f\) at the indicated point.

7. \( f(x) = -3x - 5; \quad (-2, 1) \)  
8. \( f(x) = x^2 + 10; \quad (2, 14) \)

9. \( f(x) = \sqrt{x} + 9; \quad (-5, 2) \)  
10. \( f(x) = \frac{x + 1}{x}; \quad (1, 2) \)

In Exercises 11–14, use the limit definition to find the derivative of the function.

11. \( f(x) = 7x + 3 \)  
12. \( f(x) = x^2 - 7x - 8 \)

13. \( f(x) = \frac{1}{x - 5} \)  
14. \( f(x) = \sqrt{x - 5} \)

In Exercises 15–18, find the slope of the graph of \(f\) at the indicated point.

15. \( f(x) = 8 - 5x; \quad (3, -7) \)

16. \( f(x) = -\frac{1}{2}x^2 + 2x; \quad (2, 2) \)

17. \( f(x) = \sqrt{x} + 2; \quad (9, 5) \)  
18. \( f(x) = \frac{5}{x}; \quad (1, 5) \)

In Exercises 19 and 20, use the derivative to find an equation of the tangent line to the graph of \(f\) at the indicated point.

19. \( f(x) = \frac{x^2 + 3}{x}; \quad (1, 4) \)

20. \( f(x) = -x^2 - 4x - 4; \quad (-4, -4) \)
In Exercises 21–24, determine the x-value at which the function is not differentiable.

21. \( y = \frac{x + 4}{x - 1} \)
22. \( y = -|x| + 3 \)

23. \( y = \begin{cases} -x - 2, & x \leq 0 \\ x^3 + 2, & x > 0 \end{cases} \)
24. \( y = (x + 1)^{2/3} \)

In Exercises 25–32, find the derivative of the function.

25. \( f(x) = \sqrt{x} \)
26. \( f(x) = -10^x \)
27. \( y = x^3 \)
28. \( g(x) = -\frac{1}{x^3} \)
29. \( y = \sqrt{x} \)
30. \( h(t) = \frac{1}{\sqrt{t}} \)
31. \( f(x) = 3x^4 \)
32. \( f(x) = \frac{4}{x^2} \)

In Exercises 33–38, find the equation of the tangent line at the indicated point. Then use a graphing utility to graph the function and the equation of the tangent line in the same viewing rectangle.

33. \( g(t) = \frac{2}{3t^2} \) at \((1, \frac{2}{3})\)
34. \( h(x) = \frac{2}{(3x)^2} \) at \((2, \frac{1}{18})\)

Function | Point
---|---
35. \( y = 11x^4 - 5x^2 + 1 \) | \((-1, 7)\)
36. \( y = x^3 - 5 + \frac{3}{x^3} \) | \((-1, -9)\)
37. \( f(x) = \sqrt{x} - \frac{1}{\sqrt{x}} \) | \((1, 0)\)
38. \( f(x) = 2x^{-3} + 4 - \sqrt{x} \) | \((1, 5)\)

In Exercises 39 and 40, find the average rate of change of the function over the indicated interval. Then compare the average rate of change with the instantaneous rates of change at the endpoints of the interval.

39. \( f(x) = x^2 + 3x - 4; \ [0, 1] \)
40. \( f(x) = x^3 + x; \ [-2, 2] \)

41. Revenue  The annual revenue \( R \) (in millions of dollars per year) of Reebok, International for the years 1990–1996 can be modeled by
\[
R = -1.58r^5 + 12.62r^4 + 18.12r^3 - 338.46r^2 + 922.55r + 2153.78,
\]
where \( r = 0 \) corresponds to 1990. (Source: Reebok, International)

(a) Find the average rate of change for the interval from 1992 to 1996.
(b) Find the instantaneous rate of change of the model in 1992 and 1996.
(c) Interpret the results of parts (a) and (b) in the context of the problem.

42. Cellular Telephones  The number of subscribers \( S \) (in thousands per year) of cellular telephones for the years 1988–1995 can be modeled by
\[
S = \frac{2163.9 + 666.4t}{1 - 0.18t + 0.01t^2},
\]
where \( t = 0 \) corresponds to 1988. (Source: Cellular Telecommunications Industry Association)

(a) Find the average rate of change for the interval from 1991 to 1995.
(b) Find the instantaneous rate of change of the model in 1991 and 1995.
(c) Interpret the results of parts (a) and (b) in the context of the problem.
43. Retail Price  The average retail price \( P \) in dollars of 1 pound of ground beef from 1989 to 1995 can be modeled by the equation

\[
P = 0.0025t^5 - 0.027t^4 + 0.09t^3 - 0.088t^2 - 0.067t + 2.02,
\]

where \( t \) is the year, with \( t = 0 \) representing 1990. (Source: U.S. Bureau of Labor Statistics)

(a) Find the rate of change of the price with respect to the year.
(b) At what rate is the price of beef changing in 1992? in 1995?
(c) Use a graphing utility to graph the function for \(-1 \leq t \leq 5\). During which years is the price increasing? decreasing?
(d) For what years do the slopes of the tangent lines appear to be positive? negative?
(e) Compare your answers for parts (c) and (d).

44. Recycling  The amount \( T \) of recycled paper products in millions of tons from 1970 to 1995 can be modeled by the equation

\[
T = \sqrt[3]{54.89 + 36.11t - 4.63t^2 + 0.19t^3},
\]

where \( t \) is the year, with \( t = 0 \) corresponding to 1970. (Source: Franklin Associates, Ltd)

(a) Use a graphing utility to graph the equation. Be sure to choose an appropriate window.
(c) Is \( dT/dt \) positive for \( t \geq 0 \)? Does this agree with the graph of the function? What does this tell you about this situation? Explain.

45. Velocity  A rock is dropped from a tower on the Brooklyn Bridge, 276 feet above the East River. Let \( t \) represent the time in seconds.

(a) Write a model for the position function (assume that air resistance is negligible).
(b) Find the average velocity during the first 2 seconds.
(c) Find the instantaneous velocity when \( t = 2 \) and \( t = 3 \).
(d) How long will it take for the rock to hit the water?
(e) When it hits the water, what is the rock’s speed?

46. Velocity  The straight-line distance \( s \) (in feet) traveled by an accelerating bicyclist can be modeled by

\[
s = 2r^{3/2}, \quad 0 \leq t \leq 8,
\]

where \( r \) is the time (in seconds). Complete the table showing the velocity of the bicyclist at 2-second intervals.

<table>
<thead>
<tr>
<th>Time, ( t )</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

47. Revenue, Cost, and Profit  The fixed cost of operating a small flower shop is $2500 per month. The average cost of a floral arrangement is $15 and the average price is $27.50. Write the monthly revenue, cost, and profit functions for the floral shop in terms of \( x \), the number of arrangements sold.

48. Profit  The weekly demand and cost functions for a product are

\[
p = 1.89 - 0.0083x \quad \text{and} \quad C = 21 + 0.65x.
\]

Write the profit function for this product.

Marginal Cost  In Exercises 49 and 50, find the marginal cost function.

49. \( C = 2500 + 320x \)
50. \( C = 475 + 5.25x^{2/3} \)

Marginal Revenue  In Exercises 51 and 52, find the marginal revenue function.

51. \( R = \frac{35x}{\sqrt{x} - 2}, \quad x \geq 6 \)
52. \( R = (5 + \frac{10}{\sqrt{x}}) \)

Marginal Profit  In Exercises 53 and 54, find the marginal profit function.

53. \( P = -0.0002x^3 + 6x^2 - x - 2000 \)
54. \( P = -\frac{1}{13}x^3 + 4000x^2 - 120x - 144,000 \)

In Exercises 55–74, find the derivative of the function. Simplify your result.

55. \( f(x) = x^3(5 - 3x^2) \)
56. \( y = (3x^2 + 7)(x^2 - 2x) \)
57. \( y = (4x - 3)(x^3 - 2x^2) \)
58. \( s = \left( 4 - \frac{1}{t^2} \right)(t^2 - 3t) \)
59. \( f(x) = \frac{6x - 5}{x^2 + 1} \)
60. \( f(x) = \frac{x^2 + x - 1}{x^2 - 1} \)
61. \( f(x) = (5x^2 + 2)^3 \)
62. \( f(x) = \frac{3}{x^2 - 1} \)
63. \( k(x) = \sqrt[2]{x} + 1 \)
64. \( g(x) = \sqrt{x^6 - 12x^2 + 9} \)
65. \( g(x) = x\sqrt{x^2 + 1} \)
66. \( g(t) = \frac{t}{(1 - t)^3} \)
67. \( f(x) = -2(1 - 4x^2)^2 \)
68. \( f(x) = \left( x^2 + \frac{1}{x} \right)^5 \)
69. \( b(x) = [x^2(2x + 3)]^3 \)
70. \( f(x) = [(x - 2)(x + 4)]^2 \)
71. \( f(x) = x^2(x - 1)^3 \)
72. \( f(s) = s^3(s^2 - 1)^{3/2} \)
73. \( h(t) = \frac{\sqrt{3t + 1}}{(1 - 3t)^2} \)
74. \( g(x) = \frac{(3x + 1)^2}{(x^2 + 1)^2} \)

75. **Refrigeration**  
The temperature \( T \) (in degrees Fahrenheit) of food placed in a freezer can be modeled by

\[
T = \frac{1300}{t^2 + 2t + 25},
\]
where \( t \) is the time (in hours).

(a) Find the rate of change of \( T \) when \( t = 1 \), \( t = 3 \), \( t = 5 \), and \( t = 10 \).

(b) Graph the model on a graphing utility and describe the rate at which the temperature is changing.

76. **Forestry**  
According to the Doyle Log Rule, the volume \( V \) (in board feet) of a log of length \( L \) (feet) and diameter \( D \) (inches) at the small end is

\[
V = \left( \frac{D - 4}{4} \right)^2 L.
\]

Find the rate at which the volume is changing for a 12-foot-long log whose smallest diameter is (a) 8 inches, (b) 16 inches, (c) 24 inches, and (d) 36 inches.

In Exercises 77–84, find the indicated derivative.
77. Given \( f(x) = 3x^2 + 7x + 1 \), find \( f'(x) \).
78. Given \( f(x) = 5x^4 - 6x^2 + 2x \), find \( f''(x) \).
79. Given \( f'''(x) = -\frac{6}{x^4} \), find \( f'''(x) \).
80. Given \( f(x) = \sqrt{x} \), find \( f'(x) \).
81. Given \( f(x) = 7x^{3/2} \), find \( f'(x) \).
82. Given \( f(x) = x^3 + \frac{5}{x} \), find \( f'(x) \).
83. Given \( f'(x) = 6\sqrt{x} \), find \( f''(x) \).
84. Given \( f''(x) = 20x^4 - \frac{2}{x^3} \), find \( f'''(x) \).

85. **Diver**  
A person dives from a 30-foot platform with an initial velocity of 5 feet per second (upward).

(a) Find the position function of the diver.

(b) How long will it take for the diver to hit the water?

(c) What is the diver’s velocity at impact?

(d) What is the diver’s acceleration at impact?

86. **Velocity and Acceleration**  
The position function of a particle is given by

\[
x = \frac{1}{t^2 + 2t + 1},
\]
where \( x \) is the height (in feet) and \( t \) is the time (in seconds). Find the velocity and acceleration functions.

In Exercises 87–90, use implicit differentiation to find \( \frac{dy}{dx} \).
87. \( x^2 + 3xy + y^3 = 10 \)
88. \( x^2 + 9y^2 - 4x + 3y - 7 = 0 \)
89. \( y^2 - x^2 = 49 \)
90. \( y^2 + x^2 - 6y - 2x - 5 = 0 \)

In Exercises 91–94, use implicit differentiation to find an equation of the tangent line at the indicated point.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>91. ( y^2 = x - y )</td>
<td>(2, 1)</td>
</tr>
<tr>
<td>92. ( y^2 - 3 = 2\sqrt{xy} )</td>
<td>( \left( \frac{1}{3}, 2 \right) )</td>
</tr>
<tr>
<td>93. ( 2\sqrt{x} + 3\sqrt{y} = 10 )</td>
<td>(8, 4)</td>
</tr>
<tr>
<td>94. ( y^3 - 2x^2y + 3xy^2 = -1 )</td>
<td>(0, –1)</td>
</tr>
</tbody>
</table>

95. **Water Level**  
A swimming pool is 40 feet long, 20 feet wide, 4 feet deep at the shallow end, and 9 feet deep at the deep end (see figure). Water is being pumped into the pool at the rate of 10 cubic feet per minute. How fast is the water level rising when there is 4 feet of water in the deep end?

96. **Profit**  
The demand and cost functions for a product can be modeled by \( p = 211 - 0.002x \) and \( C = 30x + 1,500,000 \), where \( x \) is the number of units produced.

(a) Write the profit function for this product.

(b) Find the marginal profit when 80,000 units are produced.

(c) Graph the profit function on a graphing utility and use the graph to determine the price you would charge for the product. Explain your reasoning.
Chapter 3 Review Exercises

In Exercises 1–4, find the critical numbers of the function.
1. \( f(x) = -x^2 + 2x + 4 \)
2. \( g(x) = (x - 1)^2(x - 3) \)
3. \( h(x) = \sqrt{x}(x - 3) \)
4. \( f(x) = (x + 1)^3 \)

In Exercises 5–8, determine the open intervals on which the function is increasing or decreasing. Solve the problem analytically and graphically.
5. \( f(x) = x^2 + x - 2 \)
6. \( g(x) = -x^2 + 7x - 12 \)
7. \( h(x) = \frac{x^2 - 3x - 4}{x - 3} \)
8. \( f(x) = -x^3 + 6x^2 - 2 \)

9. Temperature  The daily maximum temperature \( T \) (in degrees Fahrenheit) for New York City can be modeled by
\[
T = 0.036t^4 - 0.909t^3 + 5.874t^2 - 2.599t + 37.789,
\]
where \( 0 \leq t \leq 12 \) and \( t = 0 \) corresponds to January 1.
(Source: National Oceanic and Atmospheric Administration)
(a) Find the intervals in which the model is increasing.
(b) Find the intervals in which the model is decreasing.
(c) Interpret the results of parts (a) and (b).
(d) Use a graphing utility to graph the model.

10. Morning Newspapers  The number \( N \) of morning newspapers in the United States from 1970 through 1995 can be modeled by
\[
N = 0.0001t^4 - 0.01t^3 + 0.25t^2 - 1.22t + 26,
\]
where \( 0 \leq t \leq 25 \) and \( t = 0 \) corresponds to 1970.
(Source: Editor and Publisher Yearbook)
(a) Find the intervals in which the model is increasing.
(b) Find the intervals in which the model is decreasing.
(c) Interpret the results of parts (a) and (b).
(d) Use a graphing utility to graph the model.

In Exercises 11–20, use the First-Derivative Test to find the relative extrema of the function. Then use a graphing utility to confirm your result.
11. \( f(x) = 4x^3 - 6x^2 - 2 \)
12. \( f(x) = \frac{1}{4}x^4 - 8x \)
13. \( g(x) = x^2 - 16x + 12 \)
14. \( h(x) = 4 + 10x - x^2 \)
15. \( h(x) = 2x^2 - x^4 \)
16. \( s(x) = x^3 - 8x^2 + 3 \)
17. \( f(x) = \frac{6}{x^3 + 1} \)
18. \( f(x) = \frac{2}{x^2 - 1} \)
19. \( h(x) = \frac{x^2}{x - 2} \)
20. \( g(x) = x - 6\sqrt{x}, \ x > 0 \)

In Exercises 21–28, find the absolute extrema of the function on the indicated interval. Then use a graphing utility to confirm your result.
21. \( f(x) = x^2 + 5x + 6; \ [-3, 0] \)
22. \( f(x) = x^4 - 2x^3; \ [0, 2] \)
23. \( f(x) = x^3 - 12x + 1; \ [-4, 4] \)
24. \( f(x) = \frac{8}{x} + x; \ [1, 4] \)
25. \( f(x) = 3x^4 - 6x^2 + 2; \ [0, 2] \)
26. \( f(x) = -x^4 + x^2 + 2; \ [0, 2] \)
27. \( f(x) = \frac{2x}{x^2 + 1}; \ [-1, 2] \)
28. \( f(x) = 4\sqrt{x} - x^3; \ [0, 3] \)

29. Surface Area  A right circular cylinder of radius \( r \) and height \( h \) has a volume of 25 cubic inches. The surface area of the cylinder is given by
\[
S = 2\pi r \left( r + \frac{25}{\pi r} \right).
\]
Use a graphing utility to graph \( S \) and \( S' \) and find the value of \( r \) that yields the minimum surface area.

30. Waste Oxidation  When organic waste is dumped into a pond, the decomposition of the waste consumes oxygen. A model for the oxygen level \( O \) (where 1 is the normal level) of a pond as waste material oxidizes is
\[
O = \frac{t^2 - t + 1}{t^2 + 1}, \quad 0 \leq t,
\]
where \( t \) is the time in weeks.
(a) When is the oxygen level lowest? What is this level?
(b) When is the oxygen level highest? What is this level?
(c) Describe the oxygen level as \( t \) increases.
In Exercises 31–34, determine the open intervals on which the graph of the function is concave upward or concave downward. Then use a graphing utility to confirm your result.

31. \( f(x) = (x - 2)^3 \)
32. \( h(x) = x^5 - 10x^3 \)
33. \( g(x) = \frac{1}{3}(-x^4 + 8x^3 - 12) \)
34. \( h(x) = x^5 - 6x \)

In Exercises 35–38, find the points of inflection of the graph of the function.

35. \( f(x) = \frac{1}{2}x^4 - 4x^3 \)
36. \( f(x) = (x + 2)(x - 4) \)
37. \( f(x) = x^3(x - 3)^2 \)
38. \( f(x) = \frac{1}{3}x^4 - 2x^2 - x \)

In Exercises 39–42, use the Second-Derivative Test to find the relative extrema of the function.

39. \( f(x) = x^3 - 5x^3 \)
40. \( f(x) = x(x^2 - 3x - 9) \)
41. \( f(x) = (x - 1)(x + 4)^2 \)
42. \( f(x) = \frac{(x - 2)^2}{4} - x \)

In Exercises 43 and 44, identify the point of diminishing returns for the input-output function. For each function, \( R \) is the revenue (in thousands of dollars) and \( x \) is the amount spent on advertising (in thousands of dollars).

43. \( R = \frac{1}{100}(150x^2 - x^3), \quad 0 \leq x \leq 100 \)
44. \( R = \frac{2}{3}(x^3 - 12x^2 - 6), \quad 0 \leq x \leq 8 \)

45. **Minimum Sum** Find two positive numbers whose product is 169 and whose sum is a minimum. Solve the problem analytically, and use a graphing utility to solve the problem graphically.

46. **Length** The wall of a building is to be braced by a beam that must pass over a 5-foot fence that is parallel to the building and 4 feet from the building. Find the length of the shortest beam that can be used.

47. **Charitable Contributions** The percent \( P \) of income that Americans give to charities can be modeled by

\[
P = 0.0014x^2 - 0.1529x + 5.855, \quad 5 \leq x \leq 100,
\]

where \( x \) is the annual income in thousands of dollars. 
(Source: Independent Sector)

(a) What income level corresponds to the lowest percent of charitable contributions?
(b) What income level corresponds to the highest percent of charitable contributions?
(c) Use a graphing utility to verify the results of parts (a) and (b).

48. **Construction Cost** A fence is to be built to enclose a rectangular region of 4800 square feet. The fencing material along three sides costs $3 per foot. The fencing material along the fourth side costs $4 per foot.

(a) Find the most economical dimensions of the region.
(b) How would the result of part (a) change if the fencing material costs for all sides increased by $1 per foot?

49. **Tree Growth** The growth of a red oak tree is approximated by the model

\[
y = -0.003x^3 + 0.137x^2 + 0.458x - 0.839, \quad 2 \leq x \leq 34,
\]

where \( y \) is the height of the tree in feet and \( x \) is its age in years. Find the age of the tree when it is growing most rapidly. Then use a graphing utility to graph the function to confirm your result. (Hint: Use the viewing rectangle \(-10 \leq x \leq 45 \) and \(-5 \leq y \leq 60 \).)

50. **TV Usage** The average number of hours of TV usage in the United States from 1987 to 1994 can be modeled by the equation

\[
N = -0.0135t^3 + 0.457t^2 - 4.98t + 24.5,
\]

where \( t \) corresponds to 1987.

(a) Find the intervals on which \( N \) is increasing and decreasing.
(b) Find the absolute extrema on the interval \([7, 14]\).
(c) Briefly explain your results for parts (a) and (b).

51. **Palmsteiner's Law** The speed of blood that is \( r \) centimeters from the center of an artery is

\[
s(r) = c(R^2 - r^2),
\]

where \( c \) is a constant, \( R \) is the radius of the artery, and \( s \) is measured in centimeters per second. Show that the speed is a maximum at the center of an artery.

52. **Profit** The demand and cost functions for a product are

\[
p = 36 - 4x \quad \text{and} \quad C = 2x^2 + 6.
\]

(a) What level of production will produce a maximum profit?
(b) What level of production will produce a minimum average cost per unit?
53. Revenue  For groups of 20 or more, a theater determines the ticket price \( p \) according to the formula
\[
p = 15 - 0.1(n - 20), \quad 20 \leq n \leq N,
\]
where \( n \) is the number in the group. What should the value of \( N \) be? Explain your reasoning.

54. Cost  The cost of fuel to run a locomotive is proportional to the \( \frac{3}{2} \) power of the speed. At a speed of 25 miles per hour, the cost of fuel is $50 per hour. Other costs amount to $100 per hour. Find the speed that will minimize the cost per mile.

55. Inventory Cost  The cost \( C \) of inventory depends on ordering and storage costs,
\[
C = \left( \frac{Q}{x} \right) s + \left( \frac{x}{2} \right) r,
\]
where \( Q \) is the number of units sold per year, \( r \) is the cost of storing one unit for 1 year, \( s \) is the cost of placing an order, and \( x \) is the number of units in the order. Determine the order size that will minimize the cost when \( Q = 10,000 \), \( s = 4.5 \), and \( r = 5.76 \).

56. Profit  The demand and cost functions for a product are
\[
p = 600 - 3x \quad \text{and} \quad C = 0.3x^2 + 6x + 600,
\]
where \( p \) is the price per unit, \( x \) is the number of units, and \( C \) is the total cost. The profit for producing \( x \) units is
\[
P = xp - C - xx,
\]
where \( t \) is the excise tax per unit. Find the maximum profits for excise taxes of \( t = 5 \), \( t = 10 \), and \( t = 20 \).

In Exercises 57 and 58, find the intervals on which the demand is elastic, inelastic, and of unit elasticity.

57. \( p = 100 - 0.5x^2 \), \( 0 \leq x \leq 10 \sqrt{2} \)
58. \( p = \sqrt{1800 - x} \), \( 0 \leq x \leq 1800 \)

In Exercises 59–64, find the vertical and horizontal asymptotes of the graph. Then use a graphing utility to graph the function.

59. \( h(x) = \frac{2x + 3}{x - 4} \)
60. \( g(x) = \frac{5x^2}{x^2 + 2} \)
61. \( f(x) = \frac{1 - 3x}{x^2} \)
62. \( h(x) = \frac{3x}{x^2 + 2} \)
63. \( f(x) = \frac{3}{x^2 - 5x + 4} \)
64. \( h(x) = \frac{2x^2 + 3x - 5}{x - 1} \)

In Exercises 65–70, find the limit, if it exists.

65. \( \lim_{x \to 0} \left( \frac{x - 1}{x^3} \right) \)
66. \( \lim_{x \to 1} \frac{x^2 - 2x + 1}{x + 1} \)
67. \( \lim_{x \to \infty} \frac{5x^2 + 3}{2x^2 - x + 1} \)
68. \( \lim_{x \to \infty} \frac{3x^2 - 2x + 3}{x + 1} \)
69. \( \lim_{x \to \infty} \frac{3x^2}{x + 2} \)
70. \( \lim_{x \to \infty} \left( \frac{x}{x - 2} + \frac{2}{x + 2} \right) \)

71. Ultraviolet Radiation  For a person with sensitive skin, the amount \( T \) (in hours) of exposure to the sun can be modeled by
\[
T = \frac{0.37s + 23.8}{s}, \quad 0 < s \leq 120,
\]
where \( s \) is the Sunspot Scale reading. (Source: Sunspot Inc.)

(a) Use a graphing utility to graph the model. Compare your result with the graph below.
(b) Describe the value of \( T \) as \( s \) increases.

72. Average Cost and Profit  The cost and revenue functions for a product are
\[
C = 10,000 + 48.9x \quad \text{and} \quad R = 68.5x.
\]

(a) Find the average cost function.
(b) What is the limit of the average cost as \( x \) approaches infinity?
(c) Find the average profit when \( x \) is 1 million, 2 million, and 10 million.
(d) What is the limit of the average profit as \( x \) increases without bound?
In Exercises 73–80, sketch the graph of the function. Label the intercepts, relative extrema, points of inflection, and asymptotes. State the domain of the function.

73. \( f(x) = 4x - x^3 \)  
74. \( f(x) = 4x^3 - x^4 \)  
75. \( f(x) = x \sqrt{16 - x^2} \)  
76. \( f(x) = x + \frac{4}{x^2} \)  
77. \( f(x) = \frac{x + 1}{x - 1} \)  
78. \( f(x) = x^2 + \frac{2}{x} \)  
79. \( f(x) = \frac{2x}{1 + x^2} \)  
80. \( f(x) = x^{3/5} \)

In Exercises 81–84, find the differential \( dy \).

81. \( y = 6x^2 - 5 \)  
82. \( y = (3x^2 - 2)^3 \)  
83. \( y = \frac{5}{\sqrt{x}} \)  
84. \( y = \frac{2 - x}{x + 5} \)

In Exercises 85–88, use differentials to approximate the change in cost, revenue, or profit corresponding to an increase in sales of one unit.

85. \( C = 40x^2 + 1225 \), \( x = 10 \)  
86. \( C = 1.5 \sqrt{x} + 500 \), \( x = 125 \)  
87. \( R = 6.25x + 0.4x^{3/2} \), \( x = 225 \)  
88. \( P = 0.003x^2 + 0.019x - 1200 \), \( x = 750 \)

89. **Recreational Vehicle Sales**  
Sales \( S \), in billions of dollars, of recreational vehicles in the United States for the years 1980 through 1994 are given in the table.  
(Source: National Sporting Goods Association)

<table>
<thead>
<tr>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>1.2</td>
<td>1.8</td>
<td>1.7</td>
<td>3.4</td>
<td>4.1</td>
<td>3.5</td>
<td>3.9</td>
<td>4.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Year</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>4.8</td>
<td>4.5</td>
<td>4.1</td>
<td>3.6</td>
<td>4.4</td>
<td>4.8</td>
<td>4.8</td>
</tr>
</tbody>
</table>

(a) Use the regression capabilities of a graphing utility to find a quadratic model for the data, where \( t \) is the time in years, with \( t = 0 \) corresponding to 1980.  
(b) Use a graphing utility to plot the data and graph the model.  
(c) Use a graphing utility to graph \( dS/dt \).  
(d) The table shows that sales were down from 1989 through 1991. Does the derivative of the model show this decline? Explain. Give a possible explanation for the decline.

(e) Does the model show the full magnitude of the sales slump? Explain.  
(f) Use the derivative to determine the interval of time when sales were increasing most rapidly. Explain.

90. **Drug Effectiveness**  
Suppose that the effectiveness \( E \) of a pain-killing drug \( t \) hours after entering the bloodstream is  
\[ E = 22.5t + 7.5t^2 - 2.5t^3, \quad 0 \leq t \leq 4.5. \]

(a) Use a graphing utility to graph the equation. Choose an appropriate window.  
(b) Find the maximum effectiveness the pain-killing drug attains over the interval \([0, 4.5]\).

91. **Surface Area and Volume**  
The diameter of a sphere is measured to be 18 inches with a possible error of 0.05 inch. Use differentials to approximate the possible error in the surface area and the volume of the sphere.

92. **Demand**  
A company finds that the demand for its product is modeled by \( p = 85 - 0.125x \). If \( x \) changes from 7 to 8, what is the corresponding change in \( p \)? Compare the values of \( \Delta p \) and \( dp \).

93. **Economics: Revenue**  
Consider the following cost and demand information about a monopoly (in dollars).  
Complete the table, and then use the information to answer the following questions.  
(Source: Adapted from Taylor Economics, First Edition)

<table>
<thead>
<tr>
<th>Quantity of output</th>
<th>Price</th>
<th>Total revenue</th>
<th>Marginal revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>12.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>10.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>8.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>7.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>5.50</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Use the regression capabilities of a graphing utility to find a quadratic model for the total revenue data.  
(b) From the total revenue model you found in part (a), use derivatives to find an equation for the marginal revenue.  
Now use the values for output in the table and compare the results with the values in the marginal revenue column of the table. How close was your model?  
(c) What quantity maximizes total revenue for the monopoly?